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Are heavy scalars natural in minimal supergravity?

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Abstract

It has been recently claimed that very large values of a universal soft mass term m_0 for sfermions and higgs bosons become natural when M_t is close to 175 GeV if $\tan\beta \approx 10$. We show that very large values of m_0 require accidental cancellations not guaranteed by experimental data or theoretical assumptions, and consequently an unnatural fine-tuning of the parameters.

While supersymmetric particles continue to be elusive, it has been suggested that a very heavy universal scalar mass parameter m_0 should be considered 'natural', so that all sfermions and non-SM higgses could have multi-TeV masses above the LHC discovery reach. The claim is based on the observation that for values of the pole top mass M_t around its experimental value [1] and for moderately large $\tan\beta$ the MSSM RGE equations for the soft terms with minimal supergravity (mSUGRA) boundary conditions can exhibit a peculiar behavior named 'focus point' in [2]: $m_{h^{\rm u}}(Q)$ (the soft mass term of the higgs $h_{\rm u}$ coupled to up-quarks) renormalized at a scale $Q \sim$ TeV has a negligible dependence on its initial value at $Q \sim M_{\rm GUT}$.

It is easy to understand what a 'focus point' is: RGE effects trigger electro-weak symmetry breaking (EWSB) by converting a positive value of $m^2_{h^{\rm u}}(M_{\rm GUT})$ into a negative value of $m^2_{h^{\rm u}}(Q)$ if the top Yukawa coupling is sufficiently large. On the contrary, $m^2_{h^{\rm u}}(Q)$ remains positive if the top Yukawa coupling is too small. Consequently, $m^2_{h^{\rm u}}(Q)$ vanishes for some appropriate intermediate value

$$\lambda_t \sim 4\pi/\ln^{1/2}(M^2_{\rm GUT}/M^2_Z) \sim 1$$

of the top Yukawa coupling. Starting with mSUGRA boundary conditions (universal gaugino masses $m_{1/2}$ and universal scalar masses m_0 at $M_{\rm GUT} \approx 2\ 10^{16}$ GeV; for ease of illustration we assume a vanishing A_0-term), we can write

$$m^2_{h^{\rm u}}(Q) = a_0 \cdot m_0^2 + a_{1/2} \cdot m^2_{1/2}.$$

In absence of radiative corrections $a_0 = 1$ and $a_{1/2} = 0$. For an appropriate value of $\lambda_t(M_{\rm GUT})$ close to 1/2 the coefficient a_0 vanishes and a large m_0 can coexist with a small M_Z. Such a cancellation had already been noticed when the notion of fine-tuning (FT) had been introduced (see fig. 1a of [3]). If the scalar soft terms are non-universal the value of λ_t giving the analogous cancellation is different. An experimentally acceptable top mass $M_t \approx v\lambda_t \sin\beta$ can be obtained with an appropriate choice of $\tan\beta$. With universal soft terms, a_0 can vanish for moderately large values of $\tan\beta$ [2] (a regime where $\sin\beta \approx 1$ is fixed).

Unfortunately such cancellation, even if taking place, would not allow to improve the present unsatisfactory 'naturalness status' of mSUGRA models [4, 5], mainly determined by the radiative contribution to M_Z^2 proportional to the squared gluino mass M_3^2, a few times larger than M_Z^2 itself. On the contrary, the m_0^2 contribution to M_Z^2 is not problematic so that suppressing it would not help (see appendix B). However, the possibility that mSUGRA does not become less natural when $m_0 \gg M_Z$ would certainly have implications for model-building and experiments.

Is a cancellation between the m_0^2 contribution to M_Z^2 and the radiative corrections to it more 'natural' than a cancellation between different soft terms? A FT analysis says that a cancellation in the m_0^2 contribution allows to have a heavy m_0 without a large FT of the soft terms, but with a large FT of the couplings (mainly λ_t and α_3). The FT associated with the couplings is sometimes included, omitted or neglected in the various definitions of FT employed in the literature. This choice is usually irrelevant (because the fine-tuning with respect to the μ-term is often the strongest one), but not in this case. The FT-parameter used in [2] does not include the FT associated with the couplings. In the following, we will discuss why and how it has to be included, making too large values of m_0 unnatural.

The real issue does not consist of computing a number that should quantify "how much we like" the cancellation necessary to have a large m_0. The problem of 'unnatural situations' (like a strong accidental cancellation) is that they are unlikely, because they happen only in a small percentage of the available parameter space.

Consequently, in order to assess if $m_0 \gg M_Z$ is natural in minimal supergravity with $\tan\beta \approx 10$, what we should actually determine is whether the experimental determination of M_t implies that the necessary cancellation is happening i.e. if the coefficient a_0 is forced to be much smaller than its

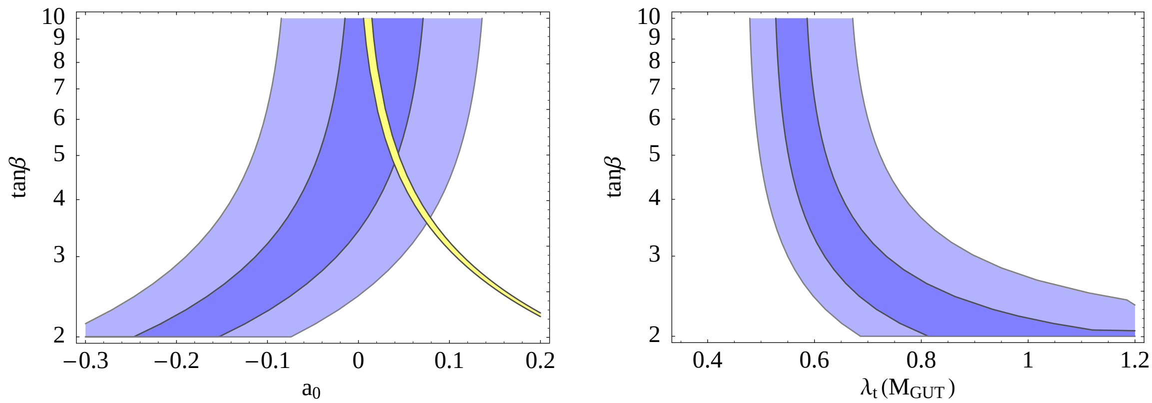

Figure 1: *The lighter regions show the uncertainties on the* $(a_0, \tan\beta)$ *(fig. 1a) and* $(\lambda_t(M_{\rm GUT}), \tan\beta)$ *(fig. 1b) plane induced by* 1σ *uncertainties on* α_3 *and* M_t *and by the uncertainty on the sparticle spectrum. In fig. 1a we have fixed* $m_0 = 3\,\mathrm{TeV}$*. Despite the top mass is precisely known, the values of* a_0 *and* $\lambda_t(M_{\rm GUT})$ *are still quite uncertain. Even if* M_t *were perfectly measured, the uncertainties would be only partially reduced (inner darker regions). Only if* a_0 *lies inside the thin light region the correction to* M_Z^2 *proportional to* m_0^2 *is smaller than* $10M_Z^2$*.*

typical value at $\tan\beta \approx 10$

$$|a_0(M_t, \alpha_i(M_Z), \text{sparticle spectrum})| \sim 0.2. \qquad (1)$$

The answer is no. The experimental uncertainty on M_t, on $\alpha_3(M_Z)$ and on the sparticle spectrum induces an uncertainty on a_0 comparable to its 'typical' value, $|a_0| \sim 0.2$, due to the strong sensitivity of a_0 to these parameters.

One way of understanding such a strong sensitivity is to neglect the threshold corrections to a_0 and to express the dependence of a_0 on the EW parameters through an integral involving the top Yukawa coupling renormalized at energies μ higher than the EW scale: $a_0 \approx 1 - \frac{3}{2}\rho$, where

$$\rho \equiv 1 - \exp\left[-6\int_{\ln m_0}^{\ln M_{\rm GUT}} \lambda_t^2(\mu)\frac{d\ln\mu}{8\pi^2}\right].$$

While M_t and $\alpha_3(M_Z)$ are known with few % uncertainty, there is a larger uncertainty on $\lambda_t(\mu)$: unknown sparticle threshold corrections affect the value of λ_t just above the SUSY breaking scale; the running up to higher scales depends on the gauge couplings (also affected by unknown sparticle threshold corrections) amplifying the uncertainties in λ_t. Of course, by solving the RGE equation for λ_t [6], ρ can be written in terms of the value of λ_t renormalized at any scale between m_0 and $M_{\rm GUT}$: for example

$$\rho = \frac{\lambda_t^2(m_0)}{E/6F} = \frac{1}{1 + [6F\lambda_t^2(M_{\rm GUT})]^{-1}}$$

where E and F are functions of the gauge couplings g_i defined as [6] $f_i(Q) \equiv \alpha_i(M_{\rm GUT})/\alpha_i(\mu)$, $E(\mu) \equiv f_1^{13/99} f_2^3 f_3^{-16/9}$, $E \equiv E(m_0) \approx 11.$ and

$$F \equiv \int_{\ln m_0}^{\ln M_{\rm GUT}} E(\mu)\frac{d\ln\mu}{8\pi^2} \approx 1.5. \qquad (2)$$

Writing a_0 in terms of $\lambda_t(m_0)$ we can estimate the uncertainty on a_0* due to the uncertainty on the couplings as

$$\delta a_0 \approx -2.5\,\delta\lambda_t(m_0) + 2.2\,\delta g_3(m_0) + \cdots.$$

Even if λ_t and g_i were measured with negligible error at the Z-scale, unknown threshold corrections would still induce a ~ 0.1 uncertainty on a_0. We illustrate this uncertainty in fig. 1a, where we show the allowed region of the $(a_0, \tan\beta)$ plane corresponding to

$$M_t = (175 \pm 5)\,\mathrm{GeV}, \qquad \alpha_3(M_Z)_{\overline{\rm MS}} = 0.120 \pm 0.003$$
$$m_0 = 3\,\mathrm{TeV}, \qquad 300\,\mathrm{GeV} < M_3 < 1\,\mathrm{TeV}$$

(lighter region). The inner darker region has been plotted assuming $M_t = 175\,\mathrm{GeV}$ in order to show that a significant uncertainty on a_0 would be present even if M_t were perfectly known. Shown are also the values of a_0 for which the m_0^2 contribution to M_Z^2 is smaller than $10M_Z^2$. For completeness, in fig. 1b we show the allowed regions of the $(\lambda_t(M_{\rm GUT}), \tan\beta)$ plane corresponding to the same parameter ranges, except for m_0, now varied between 200 GeV and 1 TeV†.

To summarize, fig. 1 shows that there is no experimental evidence that λ_t is very close to the value that gives $a_0 = 0$ — i.e. that a cancellation is suppressing the m_0^2 contribution to the Z mass. Although such a suppression is not excluded, it would require a FT of the relevant parameters inside their experimental ranges. As a consequence, m_0 can be heavy only if some cancellation is forced: between m_0^2 and the radiative corrections to it (by fine-tuning the couplings), or between m_0^2 and other soft terms (for example by fine-tuning the μ term), or both. In both cases a significant cancellation is unlikely and therefore unnatural‡.

Having explained our main point, we rediscuss it in a more quantitative way. In order to compute the naturalness upper bound on m_0 we have to estimate how unlikely is the cancellation necessary to allow large values of m_0.

*Using $\lambda_t(M_{\rm GUT})$ we would get the same uncertainty on a_0, comparable to a_0. Therefore we do not agree with J.L Feng, K.T. Mathcev and T. Moroi, hep-ph/0003138.

†We have varied the range because knowing the allowed range the top quark Yukawa coupling renormalized at the unification scale as function of $\tan\beta$ is also interesting for lepton-flavour violating signals of supersymmetric unification [7]. Fig. 1b shows in a less direct but more precise way than fig. 1a that there is no evidence for a very small value of a_0.

‡If this conclusion were not true, any supersymmetric model with very heavy sparticles could be made 'natural' provided that the soft terms depend on unmeasured couplings. Even the quantum corrections to the higgs mass in the non-supersymmetric SM could be made 'naturally' vanishing by choosing an experimentally allowed appropriate value of the SM quartic higgs coupling.

The FT parameters quantify how sensitive is M_Z with respect to variations of the parameters. Sensitivity and naturalness are however two different things [8, 9, 5]. Nevertheless, 1/FT, if much smaller than one and if divided by the 'total allowed parameter space', gives a rough measure of the percentage of the allowed parameter space where a certain cancellation happens [9]. In absence of a theoretical justification, very strong cancellations happen only in very small corners of the parameter space and are consequently very unlikely. To estimate how unlikely are the cancellations that allow a large m_0, we must therefore include the FT with respect to each relevant parameter $\wp$ and normalize it with respect to their experimentally allowed range $\Delta\wp$ [9]. More precisely, we will compute

$$\Delta(\wp) \simeq \left|\frac{\Delta\wp}{M_Z^2}\frac{\partial M_Z^2}{\partial\wp}\right| \quad \text{instead of} \quad \mathrm{FT}(\wp) \simeq \left|\frac{\wp}{M_Z^2}\frac{\partial M_Z^2}{\partial\wp}\right|$$

for each parameter $\wp$. We will then combine different FTs in the 'usual' way:

$$\Delta = \max_\wp[\Delta(\wp)].$$

although, since we want to estimate the probability that two different and almost independent cancellations could occur, it would be safe to multiply the FT parameters relative to the two cancellations, obtaining stronger bounds.

While m_0 was the only parameter considered in [2], we consider in addition the FTs with respect to variations of M_t, $\alpha_3(M_Z),\ldots$ in their experimental ranges. At present, $\Delta(M_t)$ is actually the only relevant FT besides $\Delta(m_0)$. We assume that the uncertainty on m_0^2 is comparable to m_0^2 (so that $\Delta(m_0^2) \approx \mathrm{FT}(m_0^2)$), and we (optimistically) assume a $\Delta\wp = 0.1$ total uncertainty range on $\wp = \lambda_t(M_{\rm GUT})$[§]. As observed above, one can exploit the relation between M_t and $\lambda_t(M_{\rm GUT})$ and use $\Delta(\lambda_t(M_{\rm GUT}))$ instead of $\Delta(M_t)$. The two possibilities are equivalent in the limit in which the uncertainty on M_t is the dominant one. If the error on M_t will be reduced down to a negligible level, $\Delta(\lambda_t(M_{\rm GUT}))$ will still take into account (some of) the FT associated to, e.g., α_3 so that our conclusions will still hold.

Let us discuss analytically the magnitude of $\Delta(\lambda_t(M_{\rm GUT}))$. If the experimental measure of M_t implied that $|a_0| \ll 1$, our FT-like parameter would consider as natural very large values of m_0. The variation of a_0 with $\lambda_t(M_{\rm GUT})$ is however sufficiently strong to disfavour such a possibility:

$$\mathrm{FT}(\lambda_t(M_{\rm GUT})) = \frac{36F\cdot\lambda_t^2(M_{\rm GUT})}{[1+16F\cdot\lambda_t^2(M_{\rm GUT})]^2}\frac{m_0^2}{M_Z^2}$$

where F has been defined in (2). For $\lambda_t(M_{\rm GUT})$ close to the value where the cancellation in a_0 takes place,

$$\Delta(\lambda_t(M_{\rm GUT})) \approx \frac{0.1}{\lambda_t(M_{\rm GUT})}\,\mathrm{FT}(\lambda_t(M_{\rm GUT})) \approx 0.25\frac{m_0^2}{M_Z^2}.$$

The effect of taking $\Delta(\lambda_t(M_{\rm GUT}))$ into account is shown in fig. 2. We have assumed fixed values for the gaugino masses and for the gauge couplings. For heavy m_0 there is a small portion of parameter space (limited by the dashed lines) where $\Delta(m_0^2) < 10, 30$. As explained, the smallness of this region means that there is a significant FT with respect to some other parameter. In fact this regions disappears when $\Delta(\lambda_t(M_{\rm GUT}))$ (solid line) is taken into account.

§This is the minimal uncertainty that would be obtained if M_t were known with negligible error; including the present error on M_t would make $\lambda_t(M_{\rm GUT})$ more uncertain, see fig. 1, strengthening our conclusions.

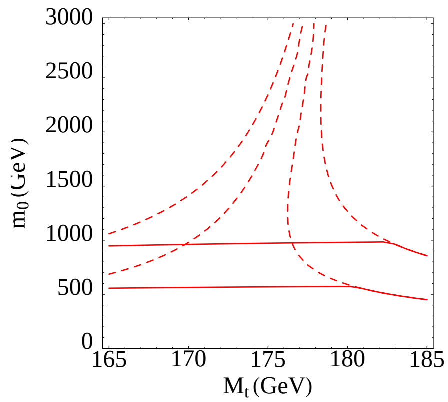

Figure 2: *Naturalness upper bounds on m_0 as a function of M_t ($\Delta < 10, 30$) before (dashed lines) and after (continuous lines) having properly taken into account the uncertainty on the relevant parameters.*

In conclusion, very heavy values of m_0 require an unnatural FT of the relevant parameters inside their present experimental range. We have used the FT-like parameter introduced in [9] and repeated the computation in appendix A using the more accurate technique presented in [5]. With respect to this problem both criteria are *less* restrictive than a 'naïve' complete FT analysis. In both cases the result is that too large values of m_0 are unnatural, as it can simply be seen by inserting a *typical* value of $|a_0|$ and the preferred confidence level on unlikely cancellations (for example $\mathrm{FT}_{\rm lim} \lesssim 1/10\%$) in the naive bound $m_0^2 \lesssim \mathrm{FT}_{\rm lim} M_Z^2/|2a_0|$. Since a_0 is typically small, eq. (1), one obtains the usual weak naturalness bound on m_0, well above all present accelerator bounds, but not above 1 TeV. Due to the smallness of $|a_0|$, the naturalness upper bound on m_0 has almost no impact on the 'naturalness status' of mSUGRA models, as discussed in appendix B.

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A Naturalness bound on m_0

As said naturalness disfavours heavy m_0 because very strong cancellations (either between different soft terms, or between the tree level m_0^2 term and the radiative corrections to it) are needed in order to accommodate very large values of m_0. Setting a naturalness upper bound on m_0 amounts to estimate how unlikely is the required cancellation in the light of our experimental and theoretical knowledge.

If we assign to the parameter space an arbitrary probability distribution function (pdf) we can compute the probability of any event, for example of the required cancellation. The pdf is however totally arbitrary in absence of experimental data. This same assumption (the choice of an arbitrary pdf, called 'Bayes prior' in statistical inference) is the crucial ingredient that allows to convert experimental data into measured ranges of fundamental parameters, like the top mass. Starting from an arbitrary pdf and using simple properties of probability, it is possible to follow how experimental data modify the probability of different values. When

experimental information is sufficiently strong, the final pdf does not depend on the arbitrary pdf needed to start with. This is why we can today assume that the pole top mass is distributed according to a 175 ± 5 gaussian.

Since the soft terms are totally unknown we assume some broad pdf for them. Our results have only a mild dependence on the pdf, unless some crazy pdf is chosen. Since M_Z (that is one combination of soft terms) has been already measured with a practically infinite precision, it is simpler to take this experimental constraint into account with the procedure used in [5]: we assume a probability distribution for the dimensionless ratios of the soft terms, and compute the overall scale of soft terms from the EWSB condition. Since in this way we never specify how heavy are the sparticles, the connection of this procedure with naturalness is quite transparent.

Sampling all parameters, like M_t and $m_0/m_{1/2}$, according to their assumed pdf, we estimated [5] that only in $p \sim 5\%$ of the cases a cancellation in the EWSB conditions generates sparticle masses above all experimental bounds in mSUGRA. In order to set upper bounds on m_0 we repeat the analysis in [5], but without averaging p over the distribution of $m_0/m_{1/2}$: we here compute p as function of $m_0/m_{1/2}$[¶] at fixed $\tan\beta = 10$. We find that $p(m_0/m_{1/2})$ has a maximum at $m_0 \sim 3m_{1/2}$, decreases when $m_0 \ll m_{1/2}$ (because too small values of m_0 give light right-handed sleptons) and becomes negligibly small when $m_0 \gg m_{1/2}$ (more precisely when $m_0 \gtrsim 3M_3$). We again conclude that values of m_0 significantly above 1 TeV require very unlikely cancellations in the EWSB condition. A certain minimal amount of cancellation is however required even for m_0 below 1 TeV in order to accommodate experimental bounds, as recalled in appendix C.

B Heavy m_0 and the naturalness problem

The Z mass is given, as function of the soft terms, by a potential minimization condition that in mSUGRA with vanishing A_0 and large $\tan\beta \approx 10$ can be approximated as

$$M_Z^2 = -2(a_0 m_0^2 + a_{1/2} m_{1/2}^2 + \mu^2). \qquad (3)$$

One important success of supersymmetry is the prediction that RGE effects typically induce negative a_i coefficients, thus establishing a direct link between SUSY-breaking and EW-breaking. This nice feature is however due to λ_t and g_3 interactions: SUSY breaking most naturally induce a non vanishing Z-boson mass comparable to the gluino and top-squark masses, that are typically heavier than the other non coloured sparticles. On the contrary experiments now tell that the Z boson is lighter than (almost) all sparticles. This kind of naturalness problem manifests itself in eq. (3) if the bounds on sparticle masses imply that the single contributions to M_Z^2 are much larger than M_Z^2 itself. What happens is that the m_0^2 contribution gives no problems, while the $m_{1/2}^2$ term gives an unpleasantly large contribution [4, 5] to M_Z^2, that can be canceled by the μ^2 term.

The m_0^2 contribution does not pose naturalness problems because the experimental bound on m_0 is weak (m_0 could even be zero), and because the coefficient a_0 is typically small, $-a_0 < 1/3$. The particular structure of the SUSY RGE protects the m_0^2 contribution from QCD corrections, that instead affect the $m_{1/2}^2$ contribution. This well known fact can be easily understood with the techniques of [10].

The $m_{1/2}^2$ term is problematic because it has a large coefficient $a_{1/2} \sim -(3 \div 5)/2$ and because LEP and Tevatron experiments provide significant lower bounds on $m_{1/2}$. The $m_{1/2}^2$ contribution to M_Z^2 is approximately given by

$$\frac{M_Z^2}{(91\,\mathrm{GeV})^2} = (5 \div 11)(\frac{M_3}{290\,\mathrm{GeV}})^2 + \cdots$$

where $M_3 \approx 2.5 m_{1/2}$ is renormalized at $Q = 500\,\mathrm{GeV}$ and lower values in the given range can be obtained for higher $\tan\beta$ and lower $\lambda_t(M_{\mathrm{GUT}})$. The LEP limit on the chargino masses gives rise, due to our assumption of gaugino mass unification, to a strong but indirect bound on the gluino mass, $M_3 \gtrsim 290\,\mathrm{GeV}$. Abandoning gaugino mass unification only the Tevatron direct bound on the gluino mass applies ($M_3 \gtrsim (180 \div 280)\,\mathrm{GeV}$, depending on the squark spectrum) so that the situation can be partially improved [11, 5]. The value of m_0 has only a small indirect impact on the naturalness problem: since $\tan\beta$ is determined by minimizing the potential, a moderately large m_0 allows to naturally obtain the moderately large values of $\tan\beta \sim 10$ for which the $m_{1/2}^2$ problem is minimized [12].

¶ We could also study p as function of m_0/M_Z. However $m_0 \gg M_Z$ is possible either because $|a_0| \ll 1$, or due to a cancellation between different soft terms. We study $p(m_0/m_{1/2})$ rather than $p(m_0/M_Z)$ because we here want to concentrate our attention on the first possibility. Bounds on m_0/M_Z have a more direct impact on phenomenology. Bounds on $m_0/m_{1/2}$ have a more direct impact on theoretical attempts of predicting $m_0/m_{1/2}$.

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